

# *Statistical Inference for Multimessenger Astrophysics*

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Columbia Experimental Gravity Group  
Columbia University in the City of New York,

and the GWHEN team

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“Pioneering MMA with GW for decades!”

 COLUMBIA UNIVERSITY  
IN THE CITY OF NEW YORK



# Multimessenger Approaches

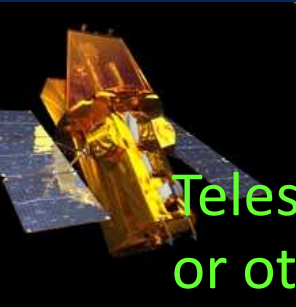
“Multi-messenger astrophysics”: connecting different kinds of observations of the same astrophysical event or system

“ExtTrig” strategy:

Telescopes, Satellites  
or other external entities

Flow of trigger  
information

GW  
Search



First astrophysically significant multimessenger result from LIGO:

Non detection of GWs from direction of GRB070201 (Andromeda galaxy) contributed to the detection of the first extragalactic SGR hyperflare

B. Abbott et al. [LIGO Collaboration], “Implications for the Origin of GRB 070201 from LIGO Observations”, *ApJ.*, 681, 1419, 2008



# Multimessenger Approaches

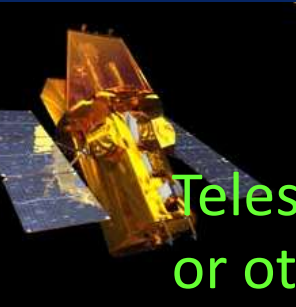
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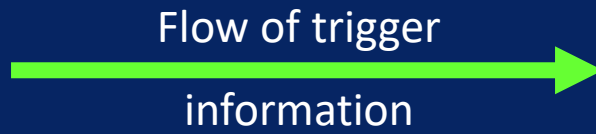
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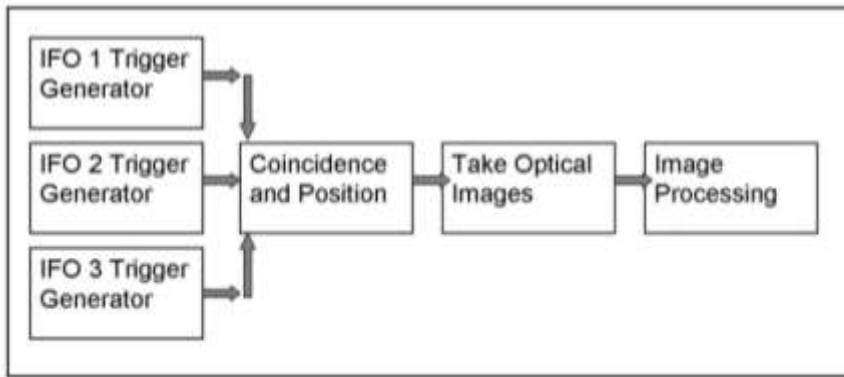


We can always go back to the collected data and analyze later

# Multimessenger Approaches: Follow-up strategy



Telescopes, Satellites  
or other external entities



**Figure 1.** A schematic of the analysis. Triggers are identified in data from the three sites of the LIGO-Virgo network. The three trigger lists are then compared to find coincident events, labeled “candidate events.” A sky region is assigned to each event candidate. The sky region is then imaged with an EM observatory. These images are searched for transients.



# Multimessenger Approaches

“Multi-messenger astrophysics”: connecting different kinds of observations of the same astrophysical event or system



GW  
Data

**“Follow-up” strategy:**

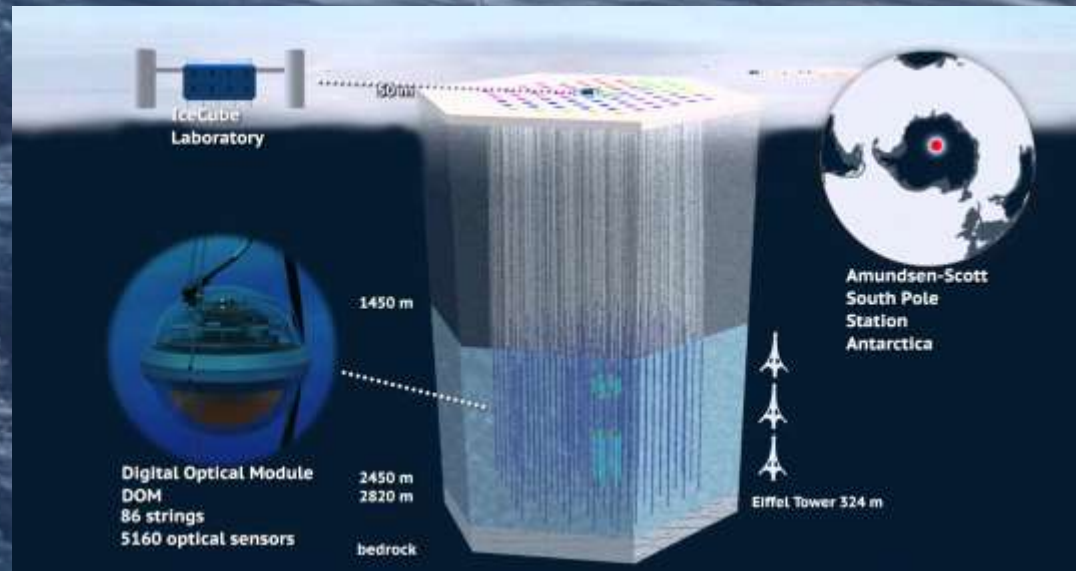
Flow of trigger  
information



Telescopes, Satellites  
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Low-latency pipeline development is essential

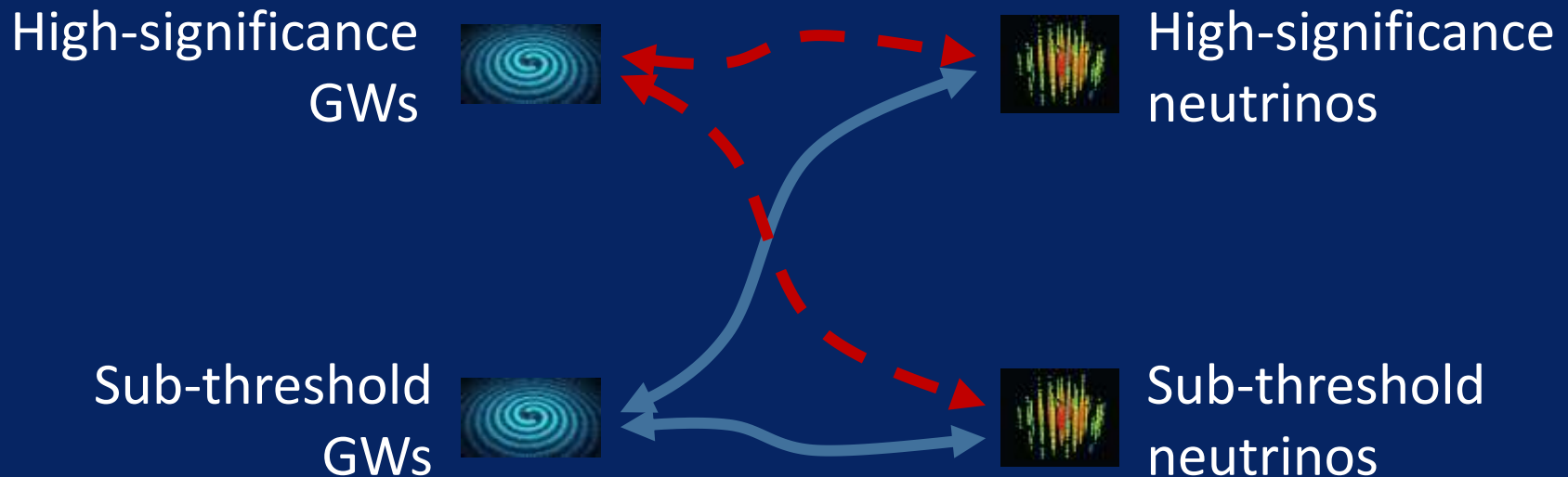


# Multimessenger Approaches

“Multi-messenger astrophysics”: connecting different kinds of observations of the same astrophysical event or system

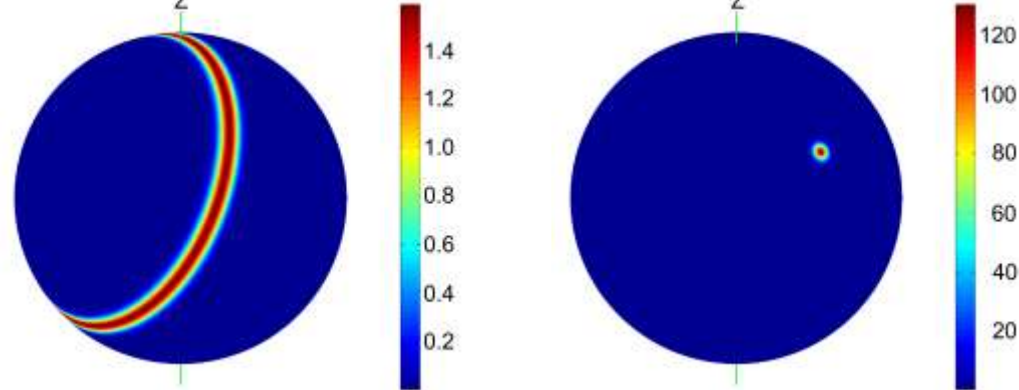
## “Joint search” strategy:

Multimessenger Search for GW+neutrino sources with complete neutrino and GW datastreams



# Multimessenger searches for GWs with LIGO: HENs

High-energy neutrino – GW multimessenger studies since 2006



"Search method for coincident events from LIGO and IceCube detectors" Class. Quantum Gravity, 25, 114039, 2008

## Astrophysics, Theory development, Method and Team building: GWHEN $\leq$ LIGO, Virgo, Icecube, ANTARES

Y. Aso et al., "Search method for coincident events from LIGO and IceCube detectors" Class. Quantum Gravity, 25, 114039, 2008

Baret et al., "Bounding the **time delay** between high-energy neutrinos and gravitational-wave transients from gamma-ray bursts", Astroparticle Physics, 35,

Ando et al., "Colloquium: Multimessenger astronomy with gravitational waves and high-energy neutrinos", Rev. Mod. Phys. 85, 1401-1420, 2013

Bartos et al., "**Observational Constraints on Multimessenger Sources of Gravitational Waves and High-Energy Neutrinos**", Physical Review Letters, 107, 251101, 2011

Baret et al., "Multimessenger Science Reach and Analysis Method for Common Sources of Gravitational Waves and High-energy Neutrinos", Physical Review D, 85, 103004, 2012

Aartsen et al., "Multimessenger search for sources of gravitational waves and high-energy neutrinos: Initial results for LIGO-Virgo and IceCube", Physical Review D, 90, 102002, 2014 (Initial LIGO/Virgo era search)

## Observational Result from O1/O2/O3

High-energy Neutrino follow-up search of Gravitational Wave Event GW150914 with ANTARES and IceCube, Antares Collaboration, IceCube Collaboration, LIGO Scientific Collaboration, Virgo Collaboration, arXiv:1602.05411, 2016

Search for high-energy neutrinos from gravitational wave event GW151226 and candidate LVT151012 with ANTARES and IceCube, Albert et al., Physical Review D, 96, 022005, 2017

Search for High-energy Neutrinos from Binary Neutron Star Merger GW170817 with ANTARES, IceCube, and the Pierre Auger Observatory, Albert et al., The Astrophysical Journal, 850, L35, 2017

Search for Multi-messenger Sources of Gravitational Waves and High-energy Neutrinos with Advanced LIGO during its first Observing Run, ANTARES and IceCube, ANTARES, IceCube, LIGO, Virgo Collaborations, Astrophys.J. 870, 134, 2019

IceCube Search for Neutrinos Coincident with Compact Binary Mergers from LIGO-Virgo's First Gravitational-Wave Transient Catalog; The Astrophysical Journal Letters, 898, L10, 2020

**Several dozens of GCNs during O2 and O3**





# GW+HEN search in low-latency aiding EM follow-up

Rapid identification is needed for timely follow-up observations of transient emission

Low-latency searches based on signals that could not individually be established as discoveries are promising

GW+HEN search is a prime motivation *for joint subthreshold search* strategy

- Both are typically emitted over a short time frame of seconds to minutes during the formation or evolution of compact objects
- Detectors searching for both messengers observe the whole sky continuously
- Joint skymap can be made rapidly available to guide follow-up electromagnetic surveys

*Proper treatment of joint event significance is essential => LLAMA*

# Basic Glossary: Multimessenger Approaches

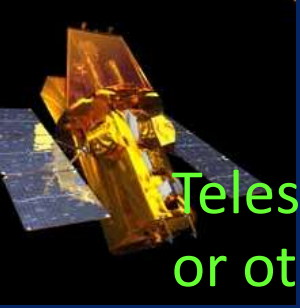
“Multi-messenger astrophysics”: connecting different kinds of observations of the same astrophysical event or system

## “ExtTrig” strategy:

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GW  
Search



## “Follow-Up” strategy:

GW  
Data

Flow of trigger  
information

Telescopes, Satellites  
or other external entities



## “Low-latency joint search” strategy:

GW detector



GRB detector



HEN detector



++



Low  
Latency  
Algorithm for  
Multimessenger  
Astrophysics



# LLAMA Employs Bayesian Method

## SEARCH INPUTS

(different for each multi-messenger trigger)

### GW

1 GW trigger

- Skymap ( $\Omega$ )
- Mean distance ( $r_{GW}$ )
- SNR ( $\rho$ )
- Time

### Neutrino

Multiple neutrino triggers

- Sky position mean (RA, Dec)
- Sky position std. dev. ( $\sigma$ )
- Energy
- Time

## SEARCH PARAMETERS

(constants used for many triggers)

- GW signal-noise ratio (SNR) background distribution
- Background neutrino triggers
- Effective IceCube area
- Integration bounds
- Empirical constants

**O2 online:** Countryman et al.; *Low-Latency Algorithm for Multi-messenger Astrophysics (LLAMA) with Gravitational-Wave and High-Energy Neutrino Candidates*; arXiv:1901.05486, 2019

**O3 online:** Keivani et al.; *Multi-messenger Gravitational-Wave + High-Energy Neutrino Searches with LIGO, Virgo and IceCube*, ICRC2019, 36, 930, 2019

**ESSENCE:** Bartos et al.; *Bayesian multimessenger search method for common sources of gravitational waves and high-energy neutrinos*; Physical Review D, 100, 083017, 2019

**Upper Limit:** Veske et al.; *Neutrino emission upper limits with maximum likelihood estimators for joint astrophysical neutrino searches with large sky localizations*; JCAP 2020

## BACKGROUND

Adjust neutrino energy window so that it matches enough background triggers

Find similar neutrinos in empirical background data

Calculate background (BG) rate

## CHANCE COINCIDENCE

Real GW, BG neutrinos

$$P_{EGW} \left( \frac{r_{GW}^2 \rho^2}{k_0^2} \right) r_{GW}^2$$

Real neutrinos, BG GW

$$\int A_{eff} \Omega_\nu d\Omega \times \int r^2 Poiss.(n, \langle n_\nu(E_\nu, r) \rangle) dr dE_\nu$$

Calculate chance coincidence likelihood

## SIGNAL

Calculate spatial overlap:

$$\int_{all\ sky} A_{eff} \Omega_\nu \Omega_{GW} d\Omega$$

Combine real GW, real neutrino probabilities with overlap integrals to get signal likelihood:

$$\int Poiss.(n, \langle n_\nu(E_\nu, r) \rangle) P(E_\nu) dE_\nu \times P_{EGW} \left( \frac{r_{GW}^2 \rho^2}{k_0^2} \right) r_{GW}^2 \times (t_\nu, t_{GW} \text{ overlap integral}) \times (\text{spatial overlap})$$

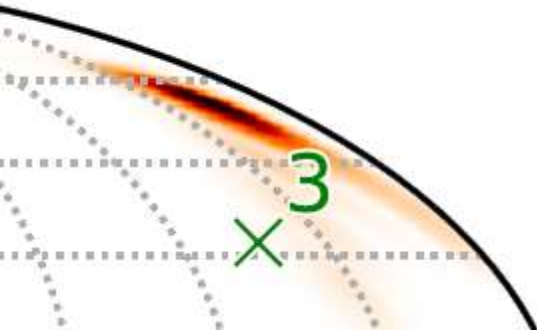
**ODDS RATIO** For any GW+HEN detection on this GW trigger (decomposed into partial fractions)

$$\sum_{i=1}^N \frac{P(H_S | x_{GW}, x_{\nu,i})}{P(H_0 | x_{GW}, x_{\nu,i}) + P(H_C | x_{GW}, x_{\nu,i})} + \sum_{i,j} \frac{P(H_S | x_{GW}, x_{\nu,i}, x_{\nu,j})}{P(H_0 | x_{GW}, x_{\nu,i}, x_{\nu,j}) + P(H_C | x_{GW}, x_{\nu,i}, x_{\nu,j})} + \dots$$

(SINGLE NEUTRINO) (DOUBLE NEUTRINO) (...)

- => Odds ratio is used as a test statistic and we perform a frequentist significance assignment
- => If **p-value > threshold**, the localization of the neutrino is sent out via GCN together with the p-value of the candidate joint GW+HEN event.
- => Otherwise and upper limit is set.

# GW+HEN alert example (S200213t)



NUMBER: 27043  
 SUBJECT: LIGO/Virgo S200213t: 1 counterpart neutrino candidate from IceCube neutrino searches  
 DATE: 20/02/13 04:40:26 GMT

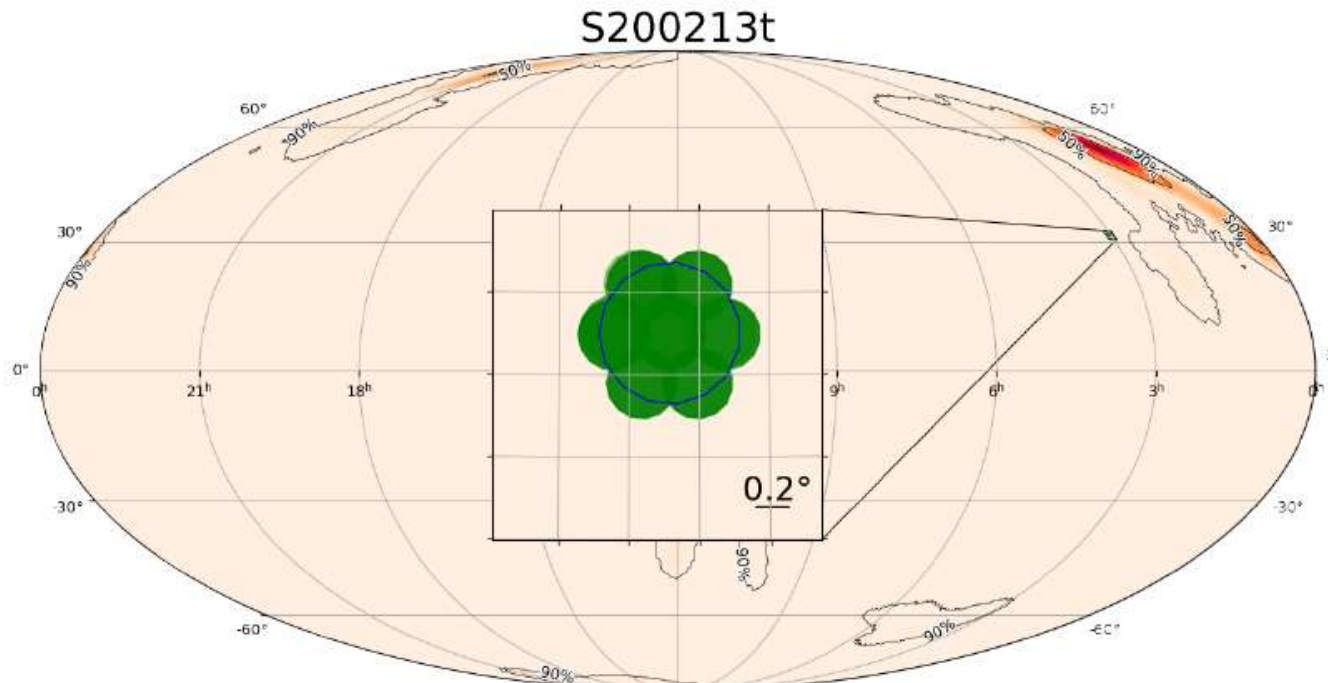
.....  
 Properties of the coincident events are shown below.

dt	ra	dec	Angular Uncertainty(deg)	p-value(generic transient)	p-value(binary merger)
-175.94	45.21	31.74	0.43	0.003	0.017

where: dt = Time offset (sec) of track event with respect to GW trigger. Angular uncertainty = Angular uncertainty of track event: the radius of a circle representing 90% CL containment by area. Pvalue = the pvalue for this specific track event from each search.

LLAMA processed all  
 Time coincident neutrinos  
 (within +/-500 s of GW trigger)  
 in 87 s.

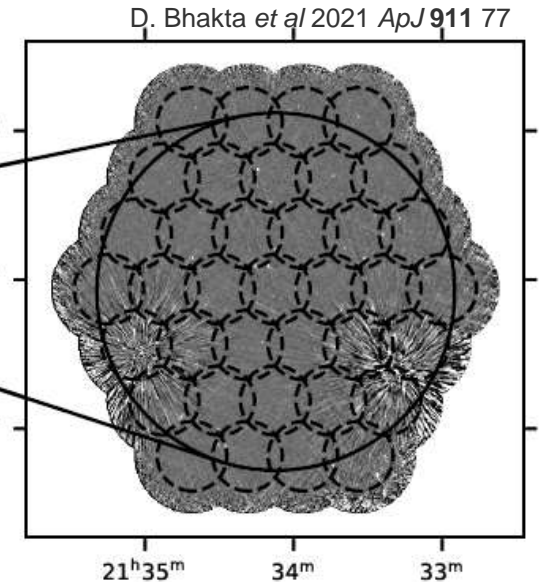
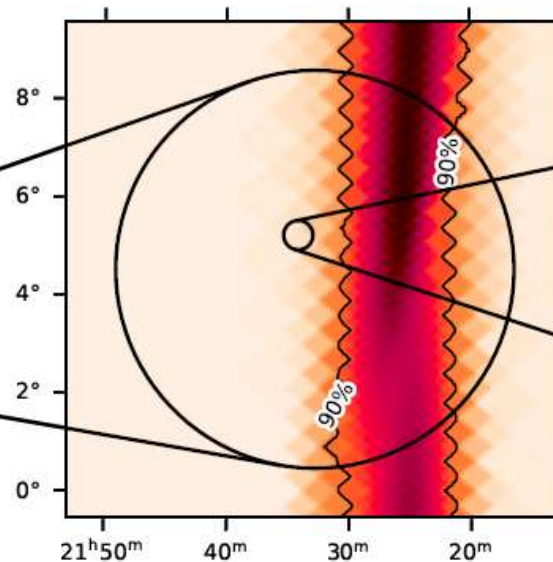
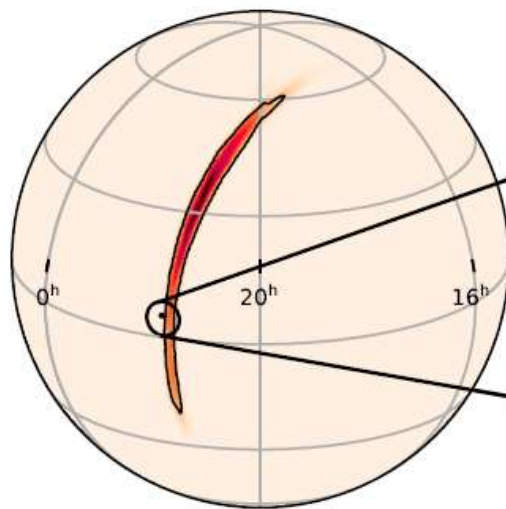
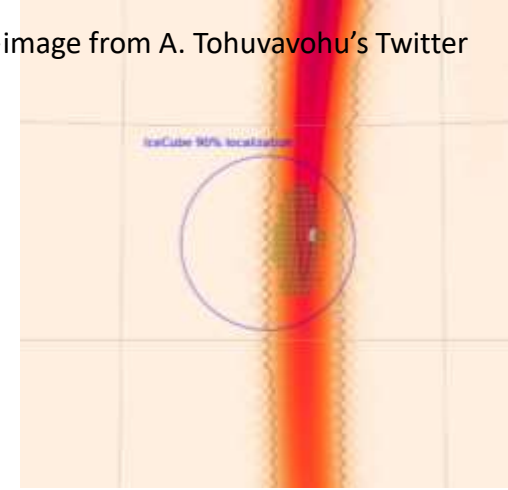
Typical event besides one p-  
 value.



# Beyond Two Messengers

Swift follow-up of S191216ap — image from A. Tohuvavohu's Twitter

- ⇒ GW candidate S191216ap by LIGO/Virgo
- ⇒ Potential neutrino counterpart from IceCube
- ⇒ HAWC subthreshold gamma ray coinciding with the GW and the neutrino on the sky
- ⇒ radio follow-up with VLA



**Need a statistical treatment for multiple messengers for such multiple coincidences!**

## Many messengers many hypotheses...

Astrophysical or noise

Related or unrelated

For  $n$  messengers, there are  $f(n+1)$  hypotheses

$$f(n) = \sum_{i=0}^{n-1} \binom{n-1}{i} f(i), \quad f(0) = 1$$

## GW+HEN+GRB case

We have 8 hypotheses (ignore unrelated two or more signal events)

- same source GW, HEN, GRB → Signal hypothesis 1
- same source GW, HEN, bg GRB → Signal hypothesis 2
- same source GW, GRB, bg HEN → Signal hypothesis 3
- same source HEN, GRB, bg GW → Signal hypothesis 4
- signal GW, bg HEN, bg GRB → BG hypothesis 1
- bg GW, signal HEN, bg GRB → BG hypothesis 2
- bg GW, bg HEN, signal GRB → BG hypothesis 3
- bg GW, bg HEN, bg GRB → BG hypothesis 4 ( $H_o$ , null)

# What is the optimal test statistic for this case?

For two hypotheses, likelihood ratio is the optimal test statistic.

*Model independent optimal multimessenger search doesn't exist!*

## Model dependent optimal test statistic with Bayesian statistics:

$$\text{TS}(\mathbf{x}) = \frac{P(\mathbf{x}|H_s)}{P(\mathbf{x}|H_n)} = \frac{\sum_i P(\mathbf{x}|H_s^i)P(H_s^i)}{\sum_j P(\mathbf{x}|H_n^j)P(H_n^j)} \times \frac{\sum_j P(H_n^j)}{\sum_i P(H_s^i)}$$

Labels in the diagram:  
 - **Combined signal hypothesis**: points to  $H_s$   
 - **Combined null hypothesis**: points to  $H_n$   
 - **Detection outcomes**: points to  $\mathbf{x}$   
 - **Model dependent prior probability**: points to  $P(H_s^i)$  and  $P(H_n^j)$   
 - **Sub-hypothesis likelihoods**: points to  $P(\mathbf{x}|H_s^i)$  and  $P(\mathbf{x}|H_n^j)$   
 - **Independent of  $\mathbf{x}$** : points to the ratio of priors  $\frac{\sum_j P(H_n^j)}{\sum_i P(H_s^i)}$

### SEARCH INPUTS

*(different for each multi-messenger trigger)*

#### GW

1 GW trigger

- Skymap ( $\Omega$ )
- Mean distance ( $r_{\text{GW}}$ )
- SNR ( $\rho$ )
- Time

#### Neutrino

Multiple neutrino triggers

- Sky position mean (RA, Dec)
- Sky position std. dev. ( $\sigma$ )
- Energy
- Time

#### GRB

1 GRB trigger

- Sky position
- Angular uncertainty
- Time
- Duration, Significance, Fluence

Common source relation through a source parameter:

$$P(\mathbf{x}|H_a^b) = \int P(\mathbf{x}|\boldsymbol{\theta}, H_a^b)P(\boldsymbol{\theta}|H_a^b)d\boldsymbol{\theta}$$

# Future Outlook

More and better quality data as a result of upgrades/new detectors

(LIGO/Virgo/KAGRA, IceCube Gen2, KM3NeT, Vera Rubin Observatory, Ultrasat, and more)



Multiple coincidences are inevitable







Statistical inference for the coinciding multiple messengers is a REQUIREMENT

**We provide a proper generalized treatment for statistical inference for multiple coincident messengers.**

It is adoptable by the Low-Latency Algorithm for Multimessenger Astrophysics pipeline (LLAMA) which is used for GW+HEN searches.



## How to Search for Multiple Messengers—A General Framework Beyond Two Messengers

Dođa Veske<sup>1</sup> , Zsuzsa Márka<sup>2</sup> , Imre Bartos<sup>3</sup> , and Szabolcs Márka<sup>1</sup> 

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